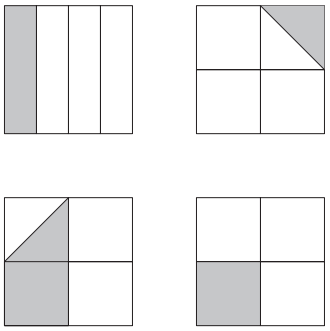
# XVI. AMC 8 Practice Questions
## Example

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?



**(A)** $12\frac{1}{2}$ **(B)** $20$ **(C)** $25$ **(D)** $33\frac{1}{3}$ **(E)** $37\frac{1}{2}$

The original problem and choices from the 2011 AMC 8 contest

**2011 AMC 8, Problem #7—**
**"Find the shaded portion of each square separately. "**

Problem number
Hint

**Solution**

**Answer (C):** The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is $25\%$ of the total area.

Solution from official solutions

**Difficulty:** Medium
**SMP-CCSS: 2, 7**
**CCSS-M: 6G.1, 6RP.3C**

Standards for Math Practice
Common Core State Standard

Difficulty, Percent correct

| | |
|---|---|
| Easy | 100-80% |
| Med Easy | 80-60% |
| Medium | 60-40% |
| Med Hard | 40-20% |
| Hard | 20-0% |

# AMC 8 Practice Questions Continued
# 12-01

Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

**(A)** $6$ **(B)** $6\frac{2}{3}$ **(C)** $7\frac{1}{2}$ **(D)** $8$ **(E)** $9$

**2012 AMC 8, Problem #1—**
**"Find how much meat the family would use."**

**Solution**

**Answer (E):** Rachelle needs $\frac{24}{8} = 3$ times the amount of meat for the picnic than she would use for her family. So she needs $3 \times 3 = 9$ pounds of meat.

OR

Set up a proportion to compare the two ratios of pounds of meat to number of hamburgers.

$$\frac{3}{8} = \frac{x}{24}$$

Solving for $x$, $8x = 72$, so $x = 9$ pounds of meat.

**Difficulty:**

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.

**CCSS-M:** 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems.

# AMC 8 Practice Questions Continued
# 12-04

Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

**(A)** $\frac{1}{24}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

**2012 AMC 8, Problem #4—**
**"Find the fraction of the total pizza for a whole slice."**

**Solution**

**Answer (C):** The whole slice that Peter ate was $\frac{1}{12}$ of the pizza. His half of the second slice was half of $\frac{1}{12}$, or $\frac{1}{24}$, of the pizza. The fraction of the pizza that Peter ate was

$$\frac{1}{12}+\frac{1}{24}=\frac{2}{24}+\frac{1}{24}=\frac{3}{24}=\frac{1}{8}.$$

**Difficulty:**
**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.
**CCSS-M:** 5.NF.1. Add and subtract fractions with unlike denominators.

# AMC 8 Practice Questions Continued
# 12-06

A rectangular photograph is placed in a frame that forms a border two inches wide on all sides of the photograph. The photograph measures 8 inches high and 10 inches wide. What is the area of the border, in square inches?

**(A)** 36 **(B)** 40 **(C)** 64 **(D)** 72 **(E)** 88

**2012 AMC 8, Problem #6—**
**"What is the width and height of the frame?"**

**Solution**

**Answer (E):**
The width of the frame is $10 + 2 + 2 = 14$ inches, and its height is $8 + 2 + 2 = 12$ inches. It encloses an area of $14 \times 12 = 168$ square inches. The photograph occupies $10 \times 8 = 80$ square inches of that area, so the area of the border itself is $168 - 80 = 88$ square inches.

**Difficulty:**
**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.
**CCSS-M:** 3.MD.7. Relate area to the operations of multiplication and addition.

# AMC 8 Practice Questions Continued
# 12-07

Isabella must take four 100-point tests in her math class. Her goal is to achieve an average grade of at least 95 on the tests. Her first two test scores were 97 and 91. After seeing her score on the third test, she realized that she could still reach her goal. What is the lowest possible score she could have made on the third test?

**(A)** $90$ **(B)** $92$ **(C)** $95$ **(D)** $96$ **(E)** $97$

**2012 AMC 8, Problem #7—**
**"What should Isabella's total score for all 4 tests be with an average of 95?"**

**Solution**

**Answer (B):**
To achieve an average grade of 95 on the four tests, Isabella must score a total of $4 \times 95 = 380$ points. She scored a total of $97 + 91 = 188$ points on her first two tests, so she must score a total of at least $380 - 188 = 192$ points on her last two tests. Because she can score at most 100 on her fourth test, she must have scored at least 92 on her third test.

OR

Isabella's first score was $95 + 2$, her second score was $95 - 4$, and her fourth score can be at most $95 + 5$. Because she can still achieve an average score of 95, her third score must have been at least $95 - 2 + 4 - 5 = 92$.

**Difficulty:**

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.

**CCSS-M:** 6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

# AMC 8 Practice Questions Continued
# 12-08

A shop advertises that everything is "half price in today's sale." In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage discount off the original price?

**(A)** 10 **(B)** 33 **(C)** 40 **(D)** 60 **(E)** 70

**2012 AMC 8, Problem #8—**
**"What is the price of an item after both discounts are applied?"**

**Solution**

**Answer (D):** The price of an item costing $\$d$ after both discounts are applied is $.80(.50d) = .40d$, a discount of 60% off the original price.

**Difficulty:**
**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.
**CCSS-M:** 7.RP.3. Use proportional relationships to solve multistep ratio and percent problems.

# AMC 8 Practice Questions Continued
# 12-09

The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

**(A)** 61 **(B)** 122 **(C)** 139 **(D)** 150 **(E)** 161

**2012 AMC 8, Problem #9—**
**"How many birds were four-legged?"**

**Solution**

**Answer (C):** All 200 heads belonged to animals with at least two legs, accounting for 400 of the 522 legs. The additional 122 legs belonged to four-legged mammals, each of which had two additional legs. So Margie saw $\frac{122}{2} = 61$ four-legged mammals and $200 - 61 = 139$ birds.

**Difficulty:**

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.

**CCSS-M:** 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

# AMC 8 Practice Questions Continued
# 12-17

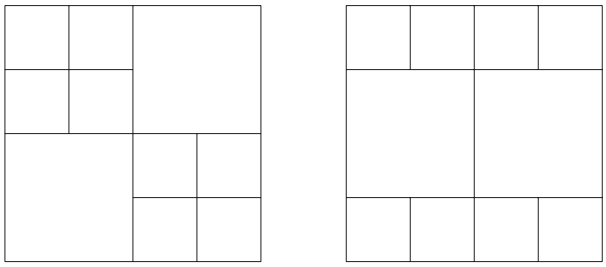
A square with an integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

**2012 AMC 8, Problem #17—**
**"What is the least possible value for the area of the original square?"**

**Solution**

**Answer (B):** The area of the original square is a square number that is more than 8, so 16 is the least possible value for the area of the original square. Its side has length 4. Two possible ways of cutting the square are shown below:



**Difficulty:**

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.

**CCSS-M:** 7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions.

# AMC 8 Practice Questions Continued
# 12-20

What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?

**(A)** $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$
**(B)** $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$
**(C)** $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$
**(D)** $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$
**(E)** $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

## 2012 AMC 8, Problem #20—
## "Use a common denominator for all three fractions."

**Solution**

**Answer (B):** Using a common denominator, $\frac{5}{19} = \frac{105}{399}$ and $\frac{7}{21} = \frac{133}{399}$, so $\frac{5}{19} < \frac{7}{21}$.
Also $\frac{7}{21} = \frac{161}{483}$ and $\frac{9}{23} = \frac{189}{483}$, so $\frac{7}{21} < \frac{9}{23}$.

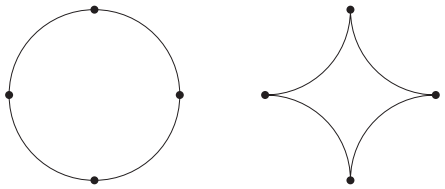
OR

Comparing each fraction, $\frac{7}{21} = \frac{1}{3}$, $\frac{5}{19} < \frac{5}{15} = \frac{1}{3}$, and $\frac{9}{23} > \frac{9}{27} = \frac{1}{3}$, so the correct increasing order is $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$.

**Difficulty:**
**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.
**CCSS-M:** 6.NS.7. Understand ordering and absolute value of rational numbers.

# AMC 8 Practice Questions Continued
# 12-24

A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



**(A)** $\frac{4-\pi}{\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** $\frac{\sqrt{2}}{\pi}$ **(D)** $\frac{\pi-1}{\pi}$ **(E)** $\frac{3}{\pi}$

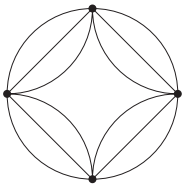
## 2012 AMC 8, Problem #24—
**"Translate the star into the circle and create a square concentric to the circle."**

**Solution**

**Answer (A):** Translate the star into the circle so that the points of the star coincide with the points on the circle. Construct four segments connecting the consecutive points of the circle and the star, creating a square concentric to the circle.

The area of the circle is $\pi(2)^2 = 4\pi$. The square is made up of four congruent right triangles with area $\frac{1}{2}(2\times 2) = 2$, so the area of the square is $4 \times 2 = 8$. The area inside the circle but outside the square is $4\pi - 8$.

This is also the area inside the square but outside the star. So, the area of the star is $8 - (4\pi - 8) = 16 - 4\pi$. The ratio of the area of the star figure to the area of the original circle is $\frac{16-4\pi}{4\pi} = \frac{4-\pi}{\pi}$.



**Difficulty:**

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them.

**CCSS-M:** 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems.